Gauge-invariant theory of two-pion photo- and electro-production off the nucleon

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Abstract A field-theoretical description of the photoproduction of two pions off the nucleon is presented that applies to real as well as virtual photons in the one-photon approximation. The Lorentz-covariant theory is complete at the level of all explicit Faddeev-type three-body final-state mechanisms of dressed interacting hadrons, including those of the nonlinear Dyson-Schwinger type. All electromagnetic currents are constructed to satisfy their respective (generalized) Ward-Takahashi identities and thus satisfy local gauge invariance as a matter of course. The Faddeev-type ordering structure results in a natural expansion of the full two-pion photoproduction current $M^\mu_{\pi\pi}$ in terms of multiple loops that preserve gauge invariance order by order in the number of loops, which in turn lends itself naturally to practical applications of increasing sophistication with increasing number of loops.

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1 Introduction

The experimental study of double-pion production off the nucleon has a fairly long history, with some of the earliest experiments going back to more than half a century. In the last two decades, with the availability of sophisticated experimental facilities at MAMI in Mainz, GRAAL in Grenoble, ELSA in Bonn, and the CLAS

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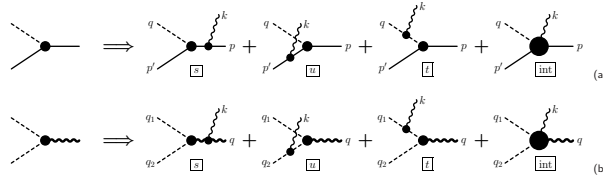E-mail: yohphy@knu.ac.kr

Fig. 1 Generic topological structures of the single-pion production current for $\gamma N \to \pi N$ and of two-pion production off a single meson, here taken to be the ρ, i.e., $\gamma \rho \to \pi\pi$, resulting from attaching a photon to the respective hadronic vertex on the left. The labels s, u, t refer to the Mandelstam variables of the respective intermediate off-shell hadron, and "int" labels the interaction currents resulting from the photon interacting with the interior of the respective vertex (which also subsume the final-state interactions of the respective outgoing hadronic two-body systems [1,2]). Time proceeds from right to left in all diagrams.

detector at JLab, the emphasis of experiments with both real and virtual photons is clearly on using this reaction as a tool to study and extract the properties of excited hadronic states that form at intermediate stages of the reaction.

Theoretically, the study of double-pion photo- and electroproduction off the nucleon is a challenging problem because, unlike single-pion production, its correct description necessarily must combine baryonic and mesonic degrees of freedom on an equal footing since the two final pions can come off a decaying intermediate meson state, and not just off intermediate baryons as a sequence of two single-pion productions. This requires accounting for all competing internal photo-subprocesses like, for example, the baryonic $\gamma N \to \pi N$ and the purely mesonic $\gamma \rho \to \pi\pi$ in a consistent manner. Moreover, this means that for double-pion production the entire field of meson spectroscopy becomes an integral part of the problem, in addition to all baryon-spectroscopic issues well known from single-pion production.

2 Formalism

We follow here the general field-theoretical approach of Haberzettl outlined in Ref. [1] for single-pion photoproduction off the nucleon, which is based on an LSZ-type reduction scheme where the photon is coupled to the connected part of the underlying hadronic Green's function by the gauge-derivative formalism [1]. The resulting generic topological structures for four-point currents like $\gamma N \to \pi N$ and $\gamma \rho \to \pi\pi$ are shown in Fig. 1. The respective interaction currents subsume the full complexity of the final-state interactions of the outgoing two-hadron systems. Practical ways to deal with this complexity are described in Ref. [2]. Using the reformulation of the production current of Ref. [3], this formalism was recently employed successfully for calculating single-pion production observables [4].

The starting points for describing the reaction $\gamma N \to \pi\pi N$ are the elementary hadronic two-pion production processes depicted in Fig. 2. For the present note, for lack of space, we will ignore here contributions arising from three- or more-pion vertices as depicted in Fig. 2(c). Their detailed treatment will be explained in a forthcoming publication [5]. Employing the elementary interactions to all orders then produces the fully dressed hadronic diagrams shown in Fig. 3. These diagrams represent the lowest orders of a three-body multiple scattering series that can be
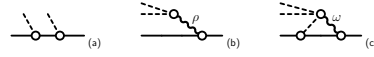
Fig. 2 Basic two-pion production processes: (a) sequential production along the nucleon line, and (b) intermediate production of a ρ-meson decaying into two pions. Part (c) provides an example of loop mechanisms based on intermediate multi-meson vertices. In addition to ρ and ω, other intermediate mesons with two-pion and three-pion decay modes, respectively, are possible. Possible contributions from five- or more-meson vertices are not shown.
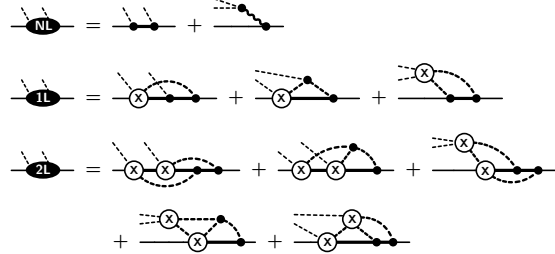

Fig. 3 Grouping of hadronic two-pion production mechanisms off the nucleon involving no loop (NL), one loop (1L), and two loops (2L). (All loops involve *fully dressed* entities.) The thick interior lines subsume all particles permitted by the process, with the solid lines indicating baryons and the dashed lines mesons. The thick wavy line stands for those mesons (like ρ, ω, etc.) that can decay into two pions (for intermediate mesons, such mesons are subsumed under the heavy dashed line). Summations over all permitted internal particles are implied. All vertices are fully dressed and the various meson-baryon or meson-meson scattering processes indicated by X are *non-polar*, i.e., they *do not* contain s-channel driving terms because their contributions are already subsumed in the full dressing of the vertices (see also [1]).
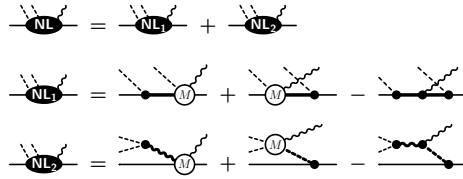

Fig. 4 Two-pion photoproduction at the no-loop level where the photon is attached to the NL diagrams of Fig. 3. The two contributions NL_1 and NL_2 correspond to the two NL diagrams in Fig. 3 in the order given. The photoproduction subamplitudes labeled M each comprise four generic terms, similar to those shown in Fig. 1. The subtractions correct the double counting resulting from the photon being attached to the respective intermediate particle in both preceding diagrams, i.e., when expanding all amplitudes M, each group consists of seven diagrams. The NL_1 and NL_2 diagram groups satisfy *independent* gauge-invariance constraints.

summed in closed form in terms of the Faddeev-type ordering structure of the Alt-Grassberger-Sandhas equations [6], amended by nonlinear driving terms that account for the fact that at intermediate stages infinitely many mesons may be produced (for full details, see [5]).

Applying now the gauge-derivative formalism [1] to the basic *dressed* hadronic processes of Fig. 3 amounts to summing up attaching the photon to these diagrams in all possible ways. For the no-loop (NL) and one-loop (NL_1) diagrams, the resulting currents are shown in Figs. 4 and 5, respectively. Higher-loop orders are not shown for lack of space. We emphasize that all input current subamplitudes appearing in Figs. 4 and 5 maintain full *local* gauge invariance, as this is necessary for any reaction theory that aims to be microscopically consistent; mere global

Fig. 5 Two-pion-production currents resulting from coupling the photon to the 1L diagrams in Fig. 3. The subtractions correct double counting of the corresponding mechanisms. For an explanation of the five-point interaction currents X^μ, see, for example, Fig. 6 in Ref. [1]. Each group $1L_i$ ($i = 1, 2, 3$) obeys an *independent* gauge-invariance constraint.

gauge invariance is not sufficient in this respect. The respective four-divergences for each of the input current pieces, therefore, provide *off-shell* generalized Ward-Takahashi identities [1]. It is shown in Refs. [1,2,3] how to do this even if some current pieces cannot be calculated exactly (which in practice invariably will be the case). Utilizing these gauge-invariant pieces as input in Figs. 4 and 5, one then shows easily that the resulting diagram groups are *separately* gauge invariant as a matter of course, with each group's four-divergence providing a generalized Ward-Takahashi identity of its own.

3 Summary

Following the field-theoretic approach of Haberzettl [1], the present note provides a diagrammatic expansion of the two-pion production current off the nucleon in terms of currents resulting from topologically distinct hadronic processes. Each current group satisfies full *local* gauge invariance provided the input currents individually satisfy their respective generalized Ward-Takahashi identities [1,2,3]. The formalism thus provides a microscopically consistent description of the reaction dynamics of two-pion production off the nucleon. Full details will be given in a forthcoming publication [5].

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